Mathematics: analysis and approaches
Higher Level
Paper 1

Name

Date: ____________________

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

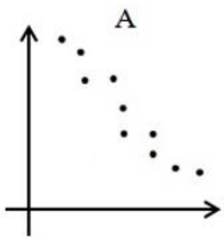
1. [Maximum mark: 6]

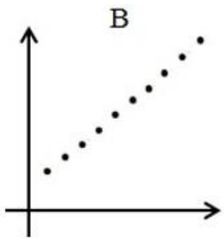
There are seven different plants being studied in a biology class. For each plant, x is the diameter of the stem in centimetres and y is the average leaf length in centimetres. Let r be the Pearson's product-moment correlation coefficient.

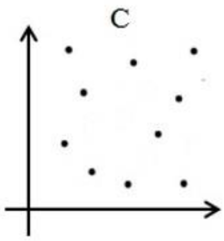
(a) Write down the possible minimum and maximum values of r. [2]

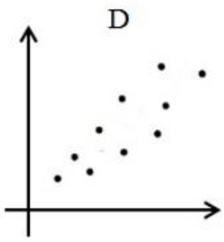
(b) Copy and complete the following table by noting which scatter diagram A, B, C, D or E corresponds to each value of r. [4]

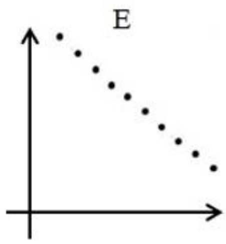
correlation coefficient r	scatter diagram
-1	
-0.8	
0	
0.5	











2. [Maximum mark: 5]

Let A and B be events such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$. Find $\mathrm{P}(A \mid B)$.

3. [Maximum mark: 5]

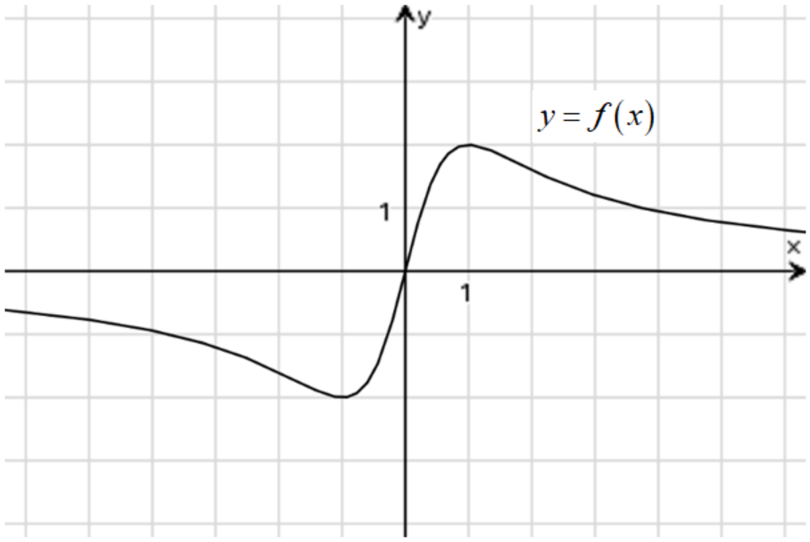
Prove that the sum of the squares of any two consecutive integers is odd.

4. [Maximum mark: 7]

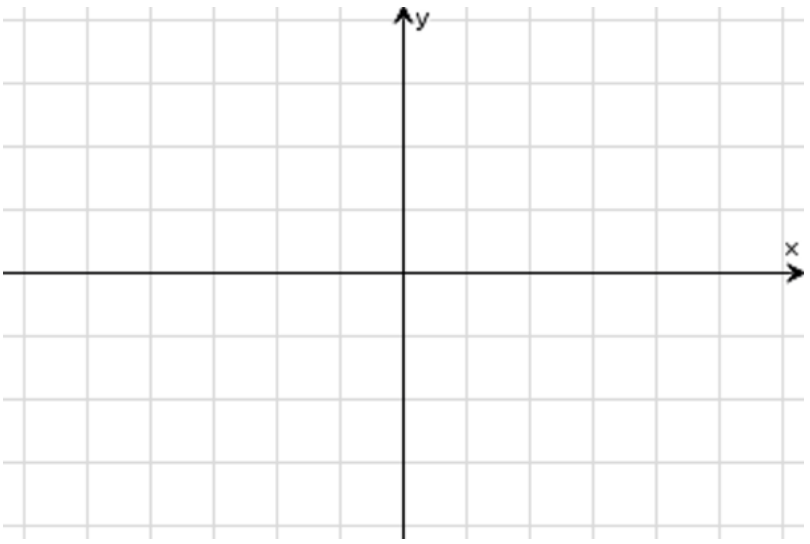
Let $g'(x) = \dfrac{2x}{\sqrt{3x^2+1}}$. Given that $g(1) = 2$, find $g(x)$.

5. [Maximum mark: 5]

The diagram below shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = 0$ (x-axis) and has a minimum at $(-1, -2)$ and a maximum at $(1, 2)$.



On the set of axes below, sketch the graph of $y = f(|x|) + 1$. Clearly show any asymptotes with their equations and the coordinates of any maxima or minima.



6. [Maximum mark: 6]

A geometric series has a common ratio of 2^x.

(a) Find the values of x for which the sum to infinity of the series exists. [2]

(b) If the first term of the series is 14 and the sum to infinity is 16, find the value of x. [4]

7. [Maximum mark: 6]

Consider the curve with the equation $x^2 - xy + y^2 = 6$.

(a) State the coordinates of all the points where the curve intersects the x-axis. [2]

(b) Find the equation for each of the two vertical lines that are tangent to the curve. [4]

8. [Maximum mark: 8]

The equation $4x^2+3x+2=0$ has roots α and β.

(a) Write down the values of $\alpha+\beta$ and $\alpha\beta$. [2]

(b) Hence, show that $\alpha^2+\beta^2=-\frac{7}{16}$. [2]

(c) Hence, find an equation with integer coefficients that has roots $2\alpha-\beta$ and $2\beta-\alpha$. [4]

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9. [Maximum mark: 8]

Using mathematical induction, prove that $n(n+1)(n+2)$ is divisible by 6 for all n, $n \in \mathbb{Z}^+$.

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 16]

The function f is defined as $f(x)=\dfrac{x+1}{\ln(x+1)}$, $x>0$.

(a) (i) Show that $f'(x)=\dfrac{\ln(x+1)-1}{\left(\ln(x+1)\right)^2}$.

(ii) Find $f''(x)$, writing it as a single rational expression [6]

(b) (i) Find the value of x satisfying the equation $f'(x)=0$.

(ii) Show that this value gives a minimum value for $f(x)$, and determine the minimum value of the function. [7]

(c) Find the x-coordinate of the one point of inflexion on the graph of f. [3]

11. [Maximum mark: 20]

The points A, B and C have position vectors $\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}$, $3\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}$ and $-\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}$ respectively and lie in plane Π.

(a) Find: (i) the area of triangle ABC;

(ii) the shortest distance from C to the line AB.

(iii) a Cartesian equation of plane Π. [14]

The line L passes through the origin and is normal to plane Π and L intersects Π at point D.

(b) Find: (i) the coordinates of point D;

(ii) the distance of Π from the origin. [6]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

(a) Find the expansion of $(\cos\theta + \mathrm{i}\sin\theta)^4$ and write it in the form $a + b\mathrm{i}$, where a and b are in terms of $\sin\theta$ and $\cos\theta$. [4]

(b) Hence, using De Moivre's theorem, show that $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$. [3]

(c) Hence, show that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$. [5]

(d) Hence, find the four solutions to $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$. [6]